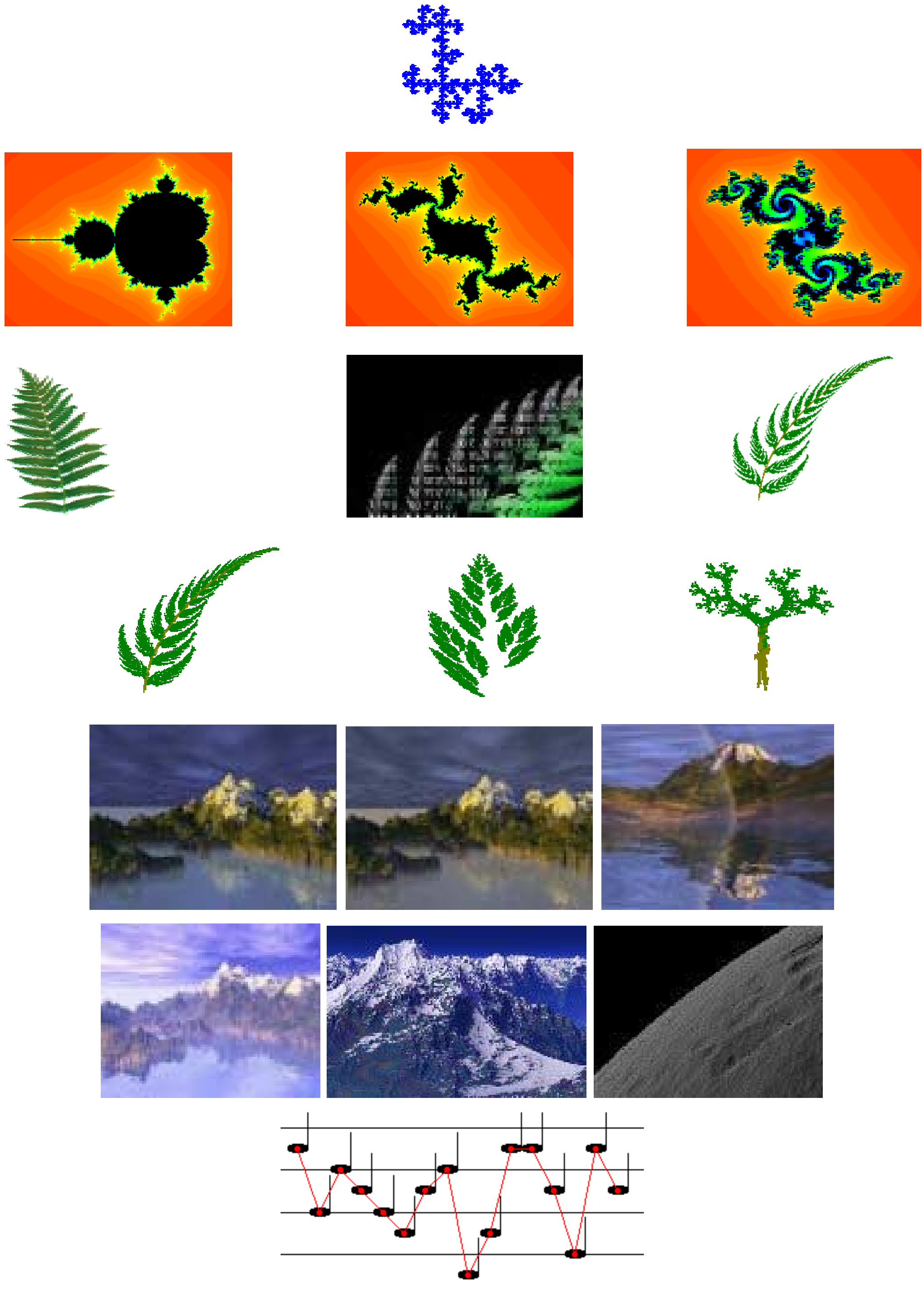
FRACTALS



Karl T. Hetland, Roberto N. Szente, Sérgio A. Boggio and Soraya S. Snarteland

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Fractals

"Is there any reason for the Geometry doesn't describe the shape of the clouds, of the mountains, of the trees or the sinuousness of the rivers? Clouds aren't spheres, mountains aren't stocks of cones, trees aren't hexagons and either the rivers draw spirals."
Benoit Mandelbrot, 1983.

"I should not ... give the impression that we have here before us a mathematician alone. Let me explain why. The first of his great insights was the discovery that the extraordinarily complex almost pathological structures, which had been long ignored, exhibited certain universal characteristics requiring a new theory of dimension to treat them adequately which he had generalised from earlier work of Hausdorff *and Besicovitch but the second great insight was that the fractal property so discovered, the general theory of which he had provided, was present almost universally in Nature. What he saw was that the overwhelming smoothness paradigm with which mathematical physics had attempted to describe Nature was radically flawed and incomplete. Fractals and pre-fractals once noticed were everywhere. They occur in physics in the description of the extraordinarily complex behaviour of some simple physical systems like the forced pendulum and in the hugely complex behaviour of turbulence and phase transition. They occur as the foundations of what is now known as chaotic systems. They occur in economics with the behaviour of prices and as Poincaré had suspected but never proved in the behaviour of the Bourse or our own Stock exchange in London. They occur in physiology in the growth of mammalian cells. Believe it or not ... they occur in gardens. Note closely and you will see a difference between the flower heads of broccoli and cauliflower, a difference that can be exactly characterised in fractal theory.* "
Benoit Mandelbrot

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Summary

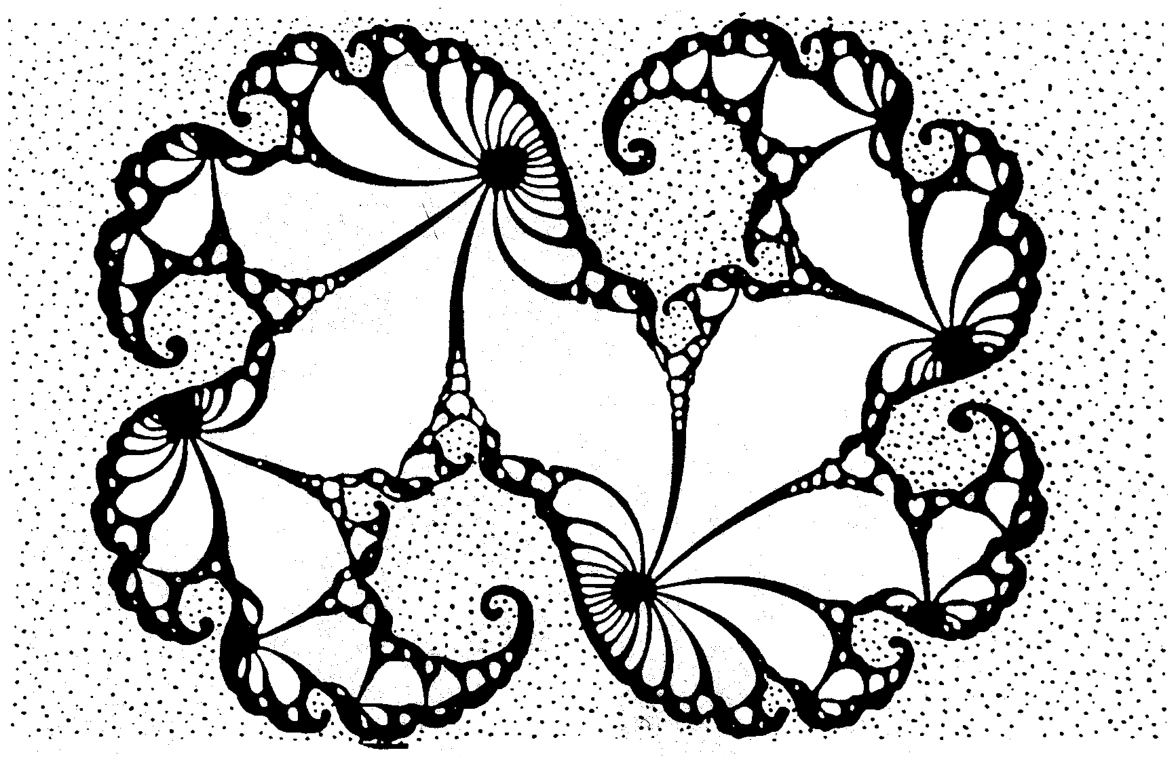
Our experience with Fractals applied in Education to the Secondary School started 6 years ago when a group of Mathematics, Physic, Chemistry and Educational Projects teachers from Colégio Bandeirantes (http://www.colband.com.br) in São Paulo, Brazil, decided to apply Fractals to the students from the 2nd grade of high school linked with Chemistry. The results and the feedbacks from them have been so good and gratifying. Here we tried to adapt it to the Norwegian students, modifying some procedures and materials from the original idea. At Bandeirantes the students has received a “Fractals kit” which they work at home, making the experiment, collecting the data and taking notes of the observations. Latter the results, data and observations are discussed during the Chemistry Lab classes.

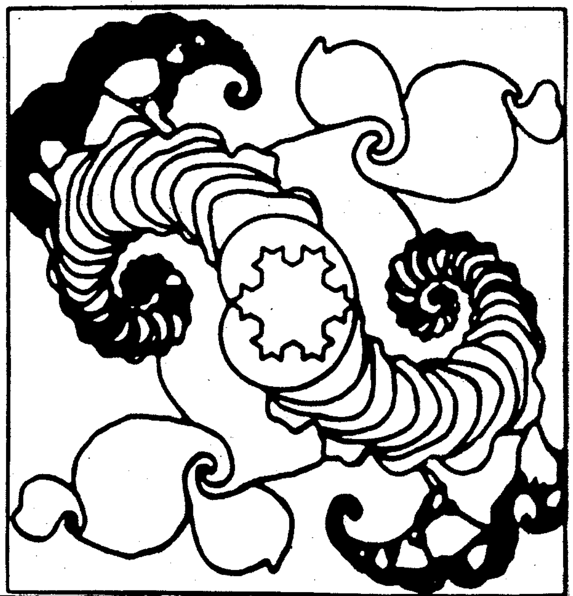
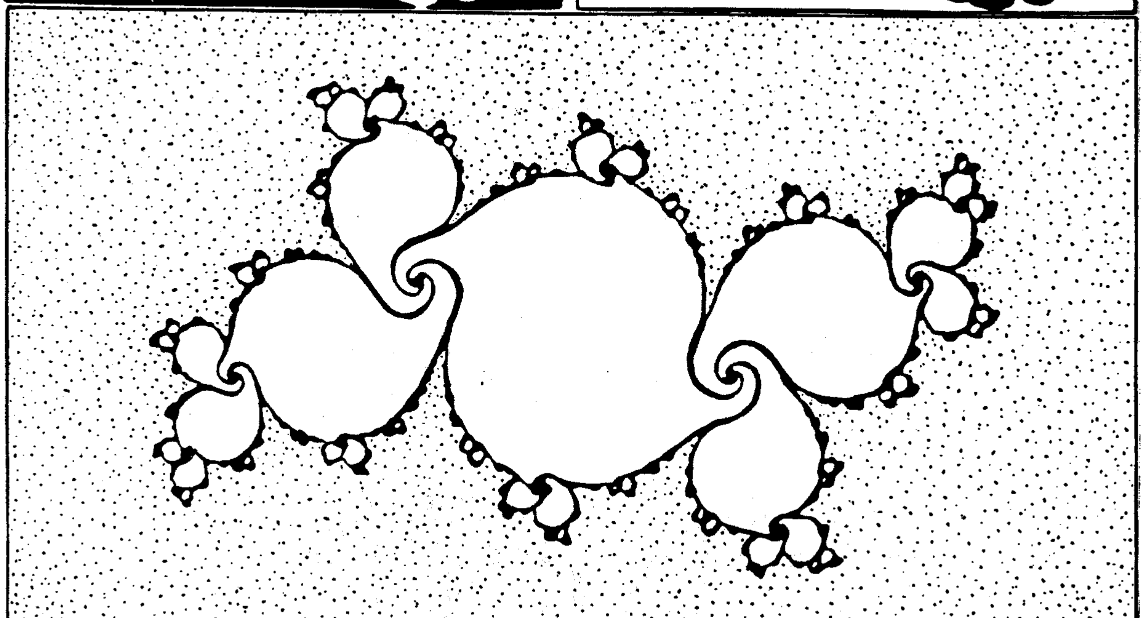
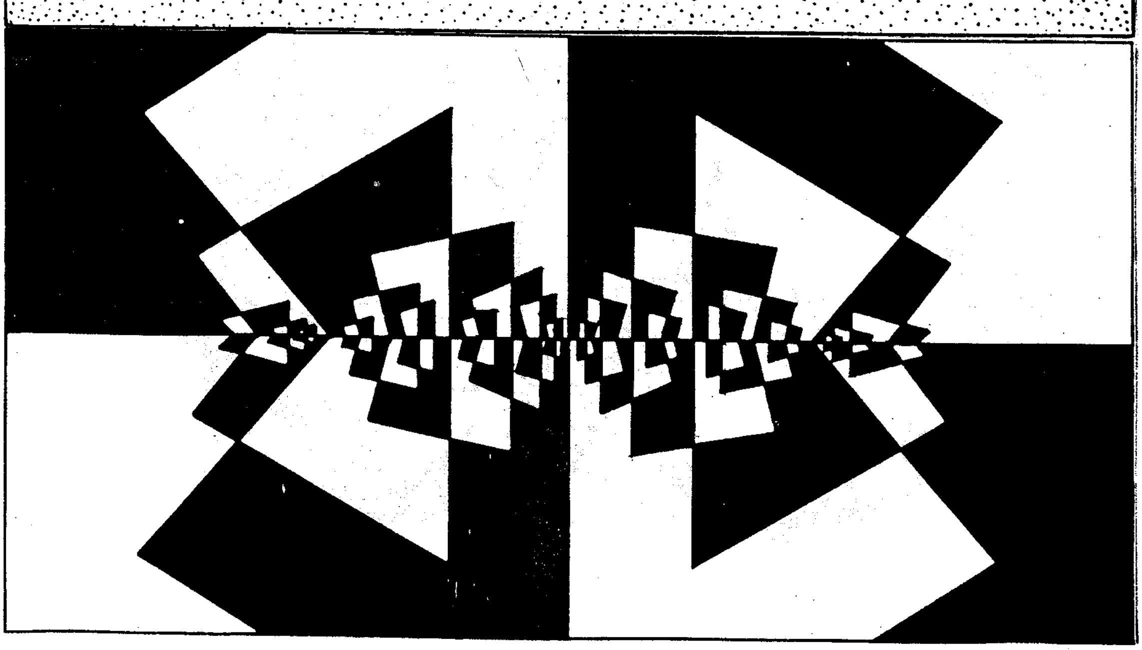
Introduction

When the people appreciate the nature, they search, most of the time, for noticing the known geometric shapes and figures. If we notice a beehive in close, it won't be difficult to identify the hexagonal shapes created, carefully, by the work of the bees. The hexagons, as the triangles and the circles, are, generally, studied in the schools, where areas, angles, and another mathematic variants of them are calculated and in this way they will became closely acquainted with these.

It is normal that, by an aesthetics reason, we use to notice, to study and to understand these shapes, which are named "perfects". A sea's star presents shapes more understanding than a clumsy octopus. In the same way, we notice the hexagonal honeycomb of the bees nearer of our knowledge than the undefined shapes of the house of a white ant. With sure, the geometry of these amorphous houses of the white ant joins another fundamental elements different of the traditional line or plane.

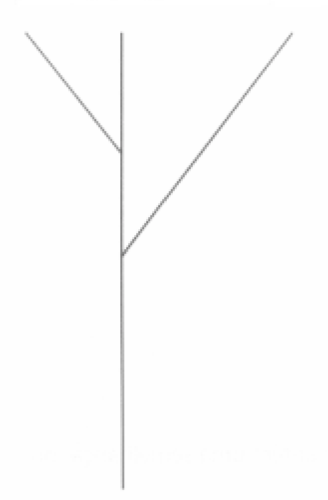
So, now a days, researches are directed to understand these undefined and random shapes, because we need to study the shapes of the clouds, of the mountains and even so, of the branches. After all, difficultly we meet an orange perfectly spherical or a trunk of bananas tree perfectly cylindrical. From the study of these undefined shapes appeared the term "Fractals" that comes from the Latin, *fractus*, which means broken or irregular.



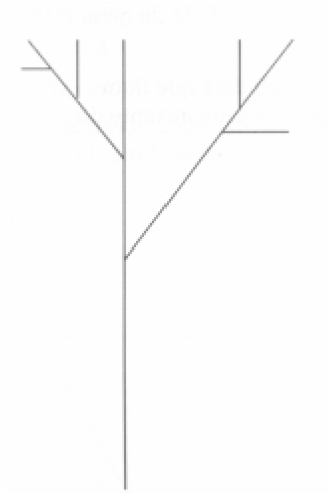
Fractal Geometry

To understand better the fractals and some terms that are characteristic of them, as “self-similar”, “recursion” and “iteration”, we can imagine a drawing by only 3 lines, a principal one and two branches; this drawing will be a rule that will be repeat and will be called seed.



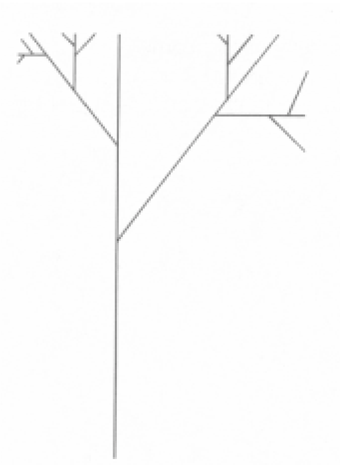
Seed

Apply this rule in the branches of the seed and we will get the first iteration of the rule.



1st iteration

Apply the rule again in all the branches from the 1st iteration and we will get the 2nd iteration of the rule.



2nd iteration

The figure that we are getting, because it is up repetition of the rule, maintains a similarity, in other words, it is **self-similar**.

The procedure of making a rule be applied in itself innumerable times is called **recursion** or **recursive process**.

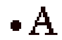
Each time this rule is applied we get a new result, or a new **iteration**.

The seed that contains the basic rule or the smaller fraction of a whole, with the capacity of generating this whole in a recursive process is the **fractal** of the whole.

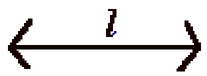
The seed of an avocado has, in its genetic code, all the rules (fractal) for generating an avocado tree (the one) by a recursive process (growing of the plant). In each moment of its growing we have iteration.

Fractal Dimension

A **point** has no dimensions - no length, no width, no height.

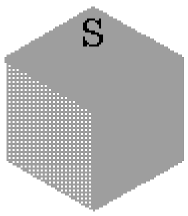


A **line** has one dimension - length. It has no width and no height, but infinite length.



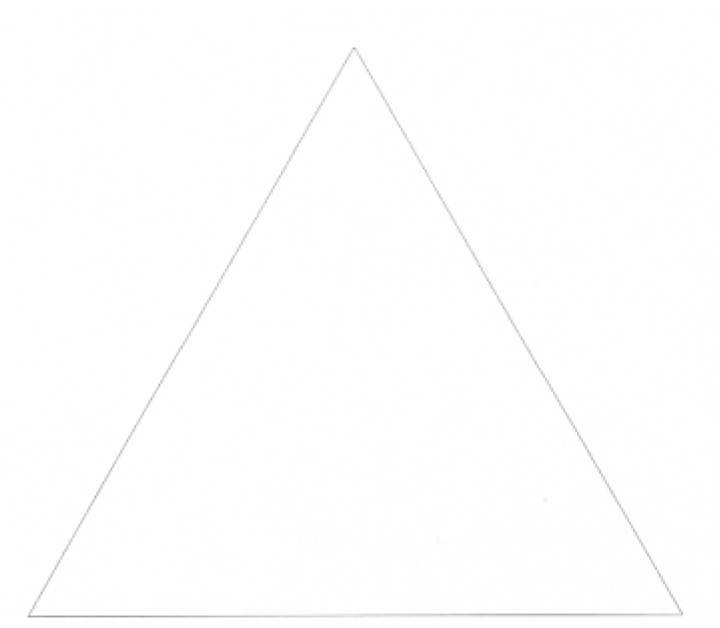
A **plane** has two dimensions - length and width, no depth. It's an absolutely flat tabletop extending out both ways to infinity.

Space, a huge empty box, has three dimensions, length, width, and depth, extending to infinity in all three directions



Fractals can have ***fractional dimension.*** A fractal might have dimension of 1.6 or 2.4. How could that be? Let's exemplify.

Exemplifying, we can use an equilateral triangle, which has 1dm in each side. The perimeter of this triangle is 3dm.

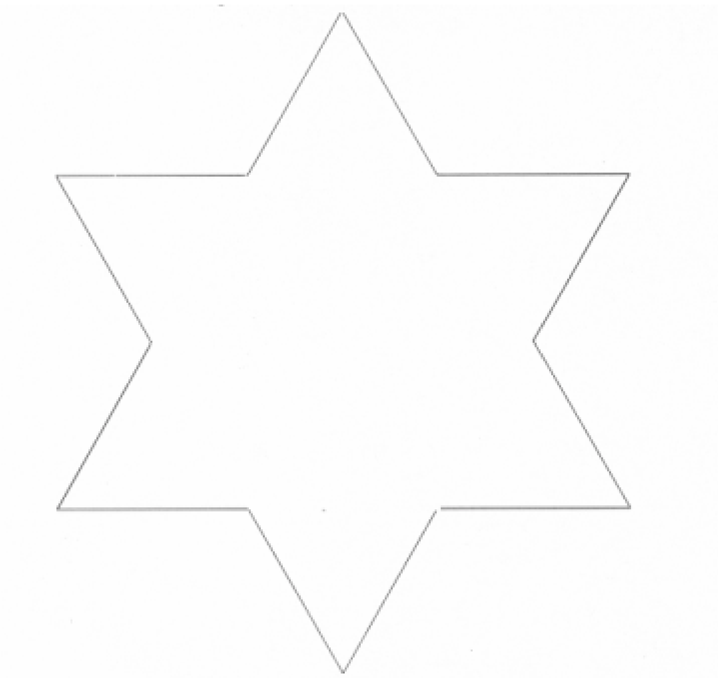


The area S of an equilateral triangle in function of its side L, can be calculated by the expression:

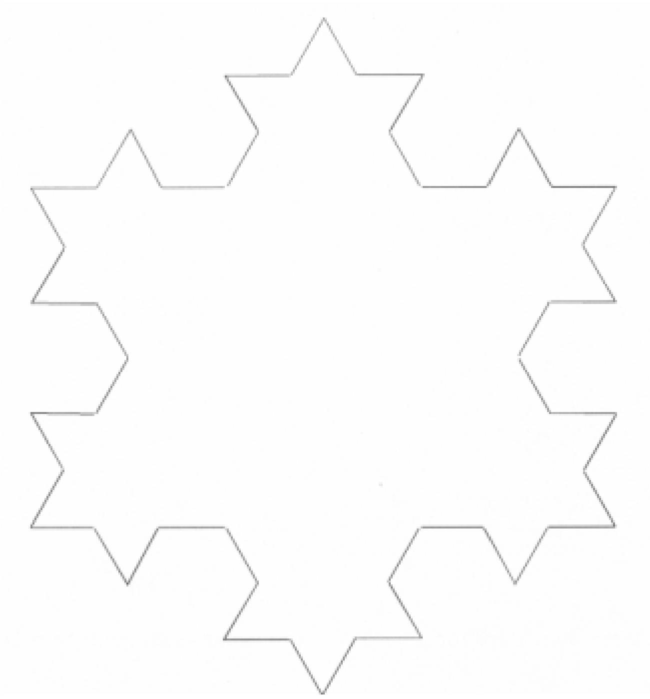
$$S = L^2 \frac{\sqrt{3}}{4}$$

In this case, $S = 0{,}4330 dm^2$

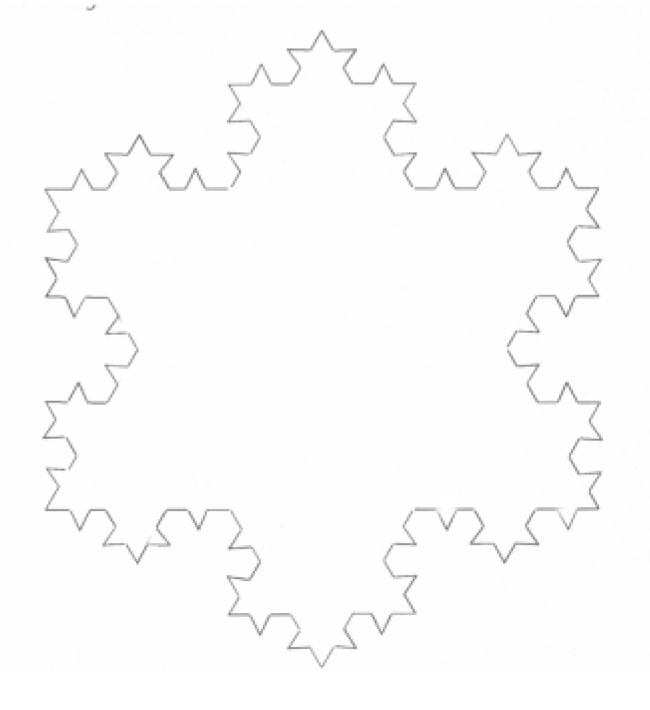
We will divide each side of the triangle in 3 equal parts, take away the central segments of each side and build an equilateral triangle over each of the parts that were taken away.



The perimeter of this new figure will be 4dm.
The total area of the figure will be:
$S = 0{,}5773 dm^2$
Repeating the previous rules we will get the following figure:



perimeter = 5,33dm
S = 0,6415dm^2
Repeating the previous rules again…



The perimeter will be 7,11dm and the area will be 0,6700dm^2.

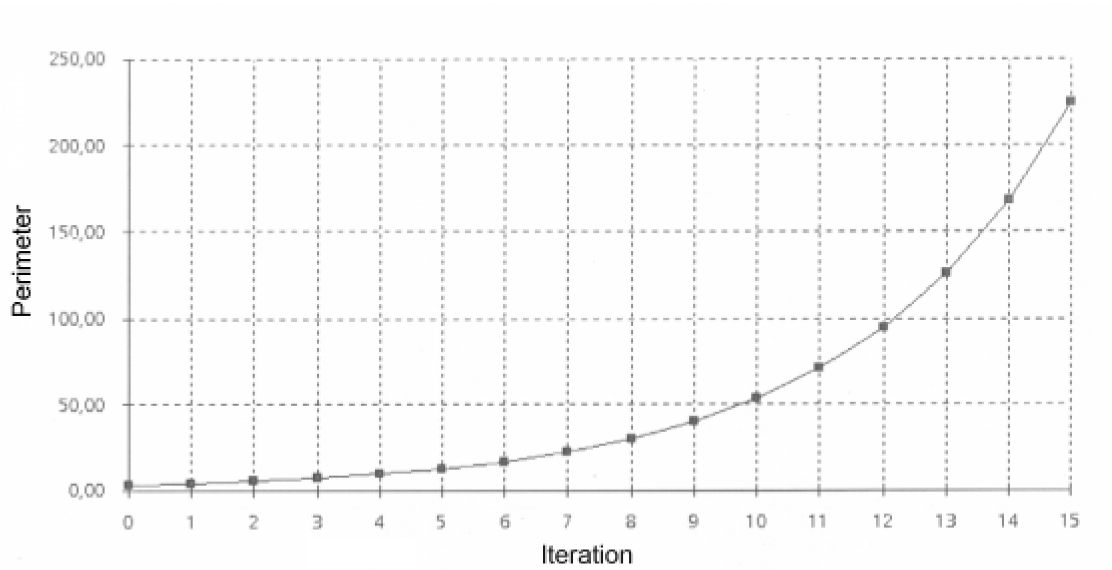
If we would repeat this process 15 times and would take notes of the data, we could build the following table of results:

Iteration	**Perimeter (dm)**	**Area (dm^2)**
O	3,00	0,4330
1	4,00	0,5773
2	5,33	0,6415
3	7,11	0,6700
4	9,48	0,6827
5	12,64	0,6883
6	16,86	0,6908
7	22,47	0,6919
8	29,97	0,6924
9	39,95	0,6926
10	53,27	0,6928
11	126,28	0,6928
12	94,71	0,6928
13	126,28	0,6928
14	168,37	0,6928
15	224,49	0,6928

This sequence of the construction of the triangles was created by Helge von Koch (1870 – 1924) and it is known as Koch Curve.

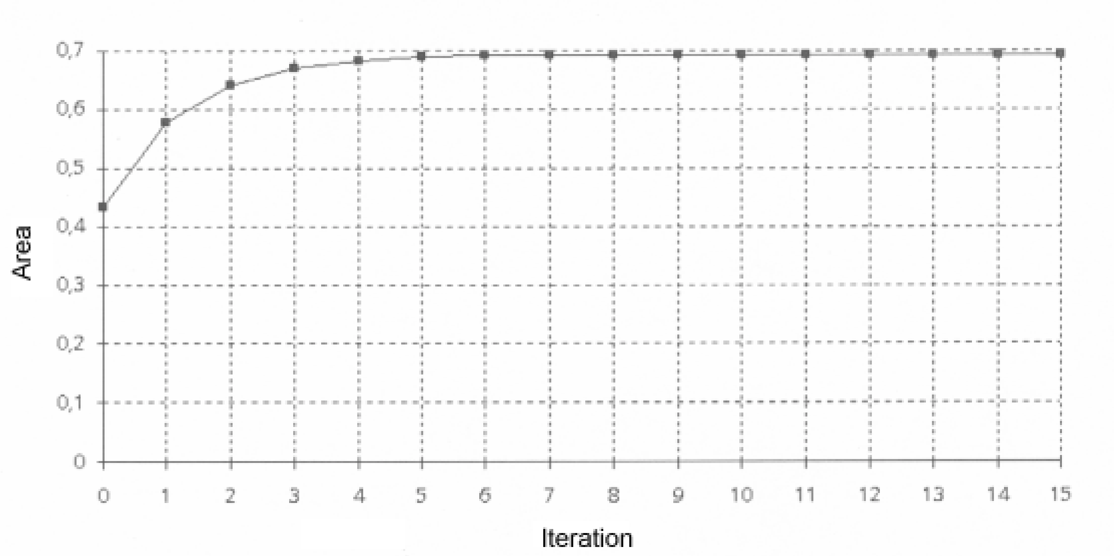
The perimeter of the figure starts in 3,00 dm and increases quickly.
In the 15^{th} iteration (application of the rules) the perimeter is 224,49 dm; it means more than 22 meters!!

Koch Curve



The area of the figure starts in 0,4330 dm^2 and decreases tending towards being constant.

Koch Curve



How is possible a figure has perimeter that increases infinitely and its internal area is finite?

As we can notice, the perimeter increases infinitely, however the internal area is finite. How will be the dimension of this figure?

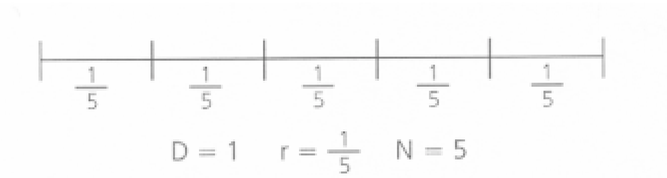
It must be bigger than 1, so it occupies the plane… but it can’t be 2 because it doesn’t occupy the plane totally.

A not whole dimension??

Yes, the dimension of this figure is approximately 1,26, and it is called **fractal dimension**.

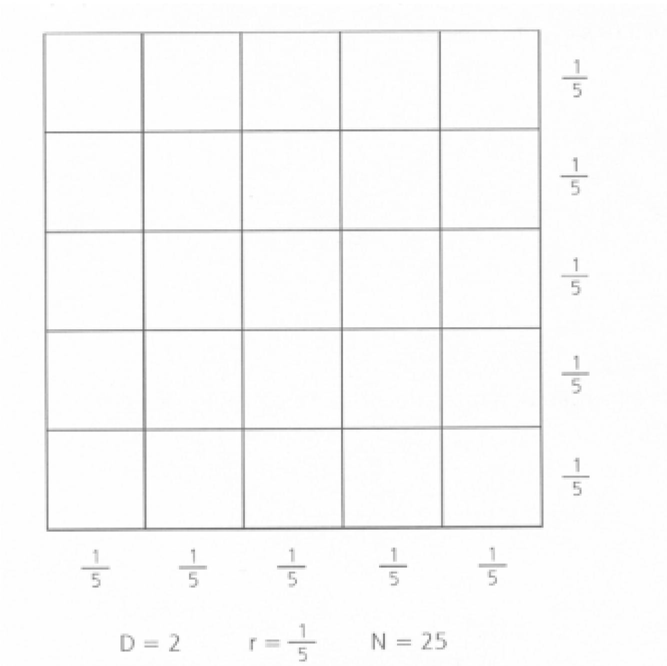
General expression to calculate of the dimension of self-similar objects.

Imagine a segment of a line divided in 5 equal parts.



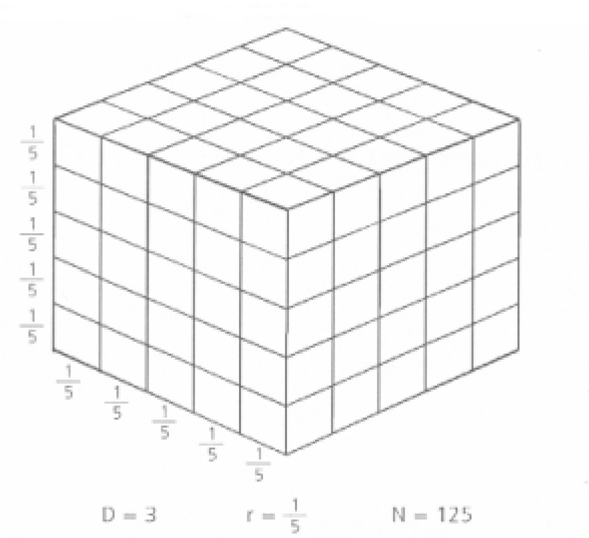
The segment of the line is a one-dimensional object (D = 1) and was divided in 5 self – similar parts (N = 5) with a decreasing ration with reference to the whole that is 1/5 (r = 1/5).

Apply the same idea in a two dimensional object (D = 2) as a square area in the plane.



Applying a decreasing ration r = 1/5, we will get N = 25 self-similar objects.
Notice that each small square is similar to the initial square, being different only by one scale or increasing factor that is 1/5.
Because of that we tell that the initial square and the small squares are self-similar.

Apply the same idea to a three dimensional object (D = 3) as a solid cube.



Applying the same decreasing ration, r = 1/5, we will get N = 125 self-similar objects.
Notice that each small cube is similar to the initial cube, being different only by a one scale or increasing factor that is 1/5.
Because of that we tell that the initial cube and the small cubes are self-similar.

In short:

D	N	N	N
1	5	5^1	$\left(\frac{1}{r}\right)^1$
2	25	5^2	$\left(\frac{1}{r}\right)^2$
3	125	5^3	$\left(\frac{1}{r}\right)^3$

We got by induction, a ration between the dimension, **D**, the scale factor, **r,** and the number of the self-similar objects, **N**.

By the Euclidean Geometry the dimension D is a whole number.

If we apply the logarithms in the expression

$$N = \left(\frac{1}{r}\right)^D$$

$$\log(N) = D \cdot \log\left(\frac{1}{r}\right)$$

$$D = \frac{\log(N)}{\log\left(\frac{1}{r}\right)}$$

we will get a general expression to the calculus of the dimension of the self-similar objects.

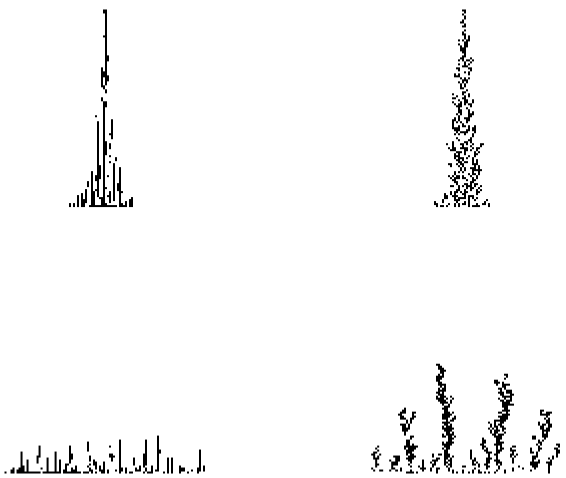
Random Iteration Algorithms

The idea of self-similarity means that if we shrink or enlarge a fractal pattern, its appearance should remain unchanged. Conversely, fractal patterns usually arise when simple patterns are transformed repetitively on smaller and smaller scales. An important class of process that produces fractal patterns are random iteration algorithms, which produce images of fractal objects. The procedure is akin to using a pen to mark dots at random on a sheet of paper. However, instead of being completely random, the movement of the pen from one position to the next is selected, at random, from a set of rules, each having fixed probability of being chosen. We can define these rules by affine transformations. An affine transformation A (in two dimensions) is a function that performs some combination of scaling, rotation, and translation on points in a plane.

Examples

Brownian Motion is an example of a process that has a fractal dimension of 2. It occurs in microscopic particles and is the result of random jostling by water molecules (if water is the medium). The path of such a particle is a “random walk” in which both direction and distance are uniformly distributed random variables. So in moving from a given location in space to any other, the path taken by the particle is almost certain to fill the whole space it reaches the exact point that is the destination.

Another aspect of Brownian motion is its effect on the formation of aggregates such as crystals. The figure down shows the structures formed under different assumptions about the relative rate of horizontal movement and the probability of a set particle sticking to fixed particles as it brushes past. “Sticky” particles tend to form structures resembling (say) trees or mosses. Such properties are exploited in animation to generate pictures of artificial plants and landscapes.



Structures arising from Brownian motion of falling particles

An example of a high fractal dimension is the internal surfaces of lung tissues. The efficiency of the lungs in diffusing oxygen from the inhaled air is directly proportional to the surface available. So for a given volume of lung it is highly advantageous to maximize the surface area. The internal surfaces of lung tissues thus have extremely high fractal dimension (about 2,9).

Electrolytic Deposition

Among the several chemicals in the nature, the metals deserve a special highlight, due to immense participation of these in different industrial processes. These metallic elements have a set of characteristic properties, as particular gleam, hardness, resistance and tenacity, in the normal conditions (except mercury, Hg, that is liquid), beyond they are good conductors of heat and electricity.

However, a specific property differentiates a metal from another, and takes part directly in the valuation or devaluation in the marketing of this metal. This property is known as “chemical reactivity” of the metal and show us why a ring of gold, silver or platinum has a so high value in the market, whereas jewellers of another metals, as zinc or copper, have so more accessible prices. In practice the results are visible. A copper ring “spoils”, in other words, rusts so easily, whereas jewels of gold use to pass on from mother to daughter. Chemically talking, the gold doesn’t rust so easily because it has a so small chemical reactivity, in other words, it is so little reactive. Now, the copper, the zinc, the iron and another metals are so more reactive than the gold, the silver or platinum, therefore they can oxide so easily, in other words, they are more inclined to rust.

In practice, we can notice clearly the difference of chemical reactivity occurred in the metals. If we add a piece of metallic zinc in a solution of copper (II) nitrate, after a certain time, we will notice that the solution will lose the original blue colour (due to the cations Cu^{2+} in solution). Moreover, the metallic zinc added will be “decreasing” each time more and more (the zinc metallic goes to the cation Zn^{2+}), while another metal will be formed into the solution (the cation Cu^{2+} goes to Cu^0).

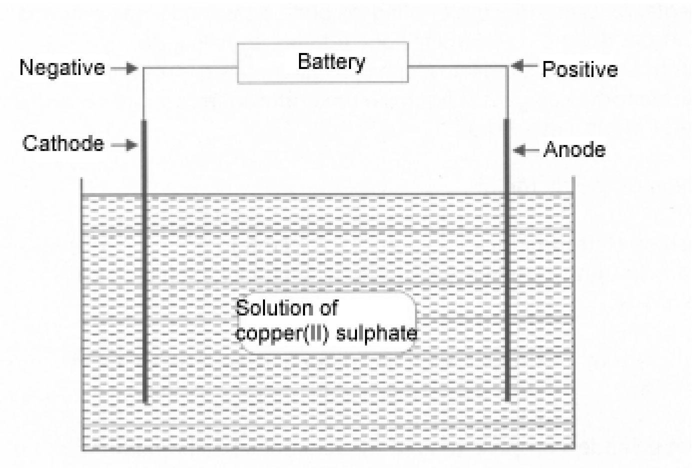
Writing the chemical equation that happened we have:

$$Zn + Cu(NO_3)_2 \longrightarrow Zn(NO_3)_2 + Cu$$

So we tell that the zinc, being more reactive than the copper, moves it from its compound.

In this way, some pieces produced with reactive metals suffer a chemical process where they receive a thin shell of a less reactive, nobler metal (gold, platinum etc), then covered with these metals the pieces will be “protected” of the corrosion process, beyond advantageously, acquire more gleam. This method of deposition of a metal over another (or sometime, over plastic and other materials) uses electricity and is known as electrolytic deposits.

To understand experimentally this process, we can notice the following scheme:



In the glass we have a solution of copper (II) sulphate ($CuSO_4$), in other words, positive ions, Cu^{2+}, and negative ions, SO_4^{2-}, separated. Immersed in this solution are placed two electrodes that are plates of metallic cooper. These electrodes are linked to an electric font of continuous current (one battery or more), in a way that one of them becomes positive and another one negative. In this way, the positive ions from de solution (Cu^{2+}) will be attracted by the negative electrode (one who has more negative electric charges). This electrode is named cathode because it attracts the cations.

So we will have an “electrolytic deposits ” of copper in the electrode (cathode).

This process can be explained chemically based on losing and getting electrons, by on metal’s part. The cation Cu^{2+} turns in metallic copper receiving (getting) two electrons. This half-reaction can be represented in the following way:

$Cu^{2+} + 2e^{-} \longrightarrow Cu^{0}$

We tell that the cation copper (II) received, got, two electrons, turning metallic copper. In this way, with the help of the electric energy, we are depositing copper over the metallic plate (in this case, metallic cooper too) immersed in the solution.

Electrolytic Deposits and Fractals

We could see some few theories about electrolytic deposits and about fractals. Now we will try to use fractals to explain some of the characteristics that happen in electrolytic metal deposits.

When we make an electrolytic metal deposits, we usually get a geometry which name is dendritical (formation of dendrites). These dendrites are like small “trees” that grew up in certain directions. The dendritical formation is strongly anisotropy, in other words, the “figure” formed in the end of the electrolytic metal deposits isn’t homogeneous in the plane (or space), but presents itself in some positions forming the dendrites.

As it was mentioned before, an electrolytic metal deposits depends of the concentration of the solution and of the voltage (and applied current). For certain concentrations and voltages, the shape of the electrolytic deposits changes itself from dendritical to another isotropic shape, where it isn’t noticed a main orientation in the formed figure. This isotropic shape has fractal characteristics, and can be studied with the help of this tool. The concentrations and voltages where these changing of the shape from dendritical to fractal happen are function of the kind of salt (metal) that we are working with.

Summarising what was mentioned, fractal shapes in the electrolytic metal deposits depend on the experimental conditions. The process of forming these figures is an irreversible diffusion of metallic ions in solution until they are incorporated in the structure.

Now, some more about electrolytic metal deposits theory.

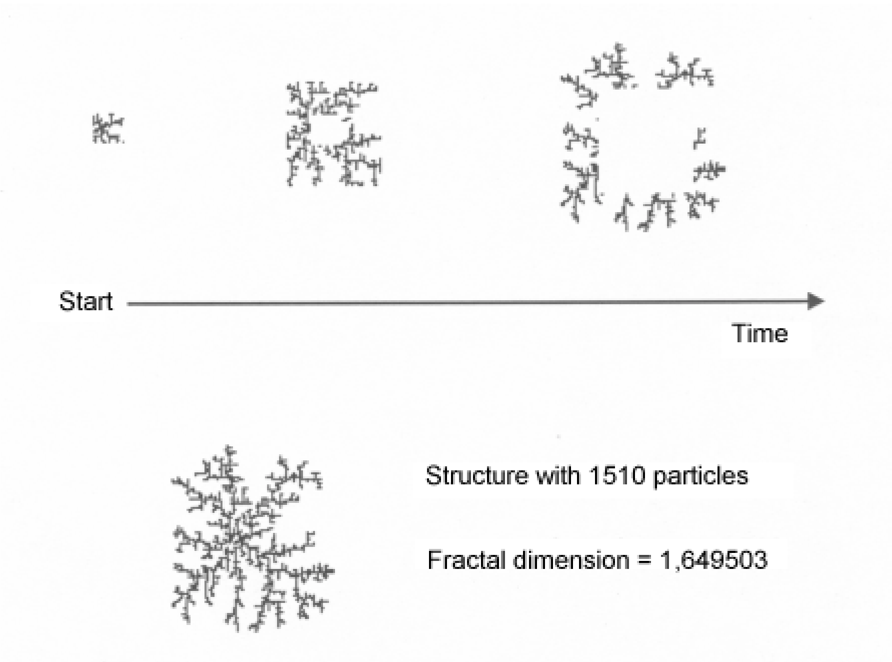
As we have mentioned before, the electrolytic metal deposits process is ruled by a diffusive process. The equation that usually “translates” this kind of process is the differential equation of Laplace with the conditions of movable interfaces.

Does it sound more or less complicated?

It is true, this kind of equation can become extremely complex to be solved and the results got are extremely difficult to be interpreted.

In 1981, two researchers (Witten and Sander) proposed a model that they called Diffusion Limited Aggregation (DLA). This is a model that allows the simulation of electrolytic deposits in a so easy way, as we will see next.

In the DLA model the rules are simple: one particle is placed in the centre of an electrolytic cell that is being studied. Another one is threw far from the origin and it is allowed “to walk” random (without a main direction) until the particle arrives near of the first one. In this moment the particle stops and becomes part of the deposited structure. So a new particle is thrown and the process is repeated.



Using this model, we get figures that present characteristics very similar to that one found experimentally in electrolytic metal deposits in certain experimental conditions. Some of these characteristics found are exactly of a fractal structure (fractional, self-similar and microscopic details).

Experimental Part

Assembly of the generator plate of fractals

Necessary materials

1 plastic Petre (internal diameter≈9cm)
1 glass Petre (internal diameter=9cm)
1 results sheet
1 square paper
60 cm copper wire
10 mL $CuSO_4$ 1M
Battery
Metallic tongs
Butanbrenner

Procedure

Attention: All the experiments must be made in a place where you have a table and water near.

How we will deal with chemicals, it is advisable to dress **old clothes or apron**.

Use old duster or old tea towel to clean and dry the objects and the hands.

Take the plastic Petre and put it over the square paper.

With a pen draw the center of the circles in the plastic Petre.

Take the Butanbrenner, a metalic tongs and a piece of copper wire. Ligth the Butanbrenner with care (use the correct technique) and with the tongs hold on the copper wire and warm it in the flame. After around 1 minute try to put the wire through of the center in the plastic Petre.



Now we have the plastic Petre with copper wire in the center. It will be the lid of the glass Petre.

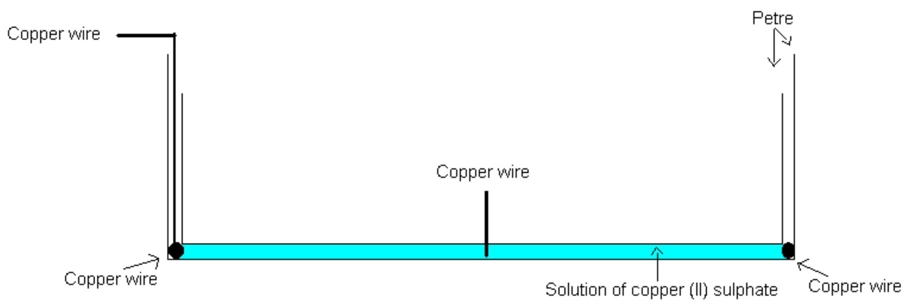
Take 60cm of copper wire and make a ring with that, leaving around 2 or 3 cm in the end out of the ring (it will be used to connect this electrode with the positive pole of a battery).

Take the glass Petre and put the copper ring inside it.



Pour softly around 10ml of $CuSO_4$ 1M into the glass Petre.

Put the plastic Petre, wich has the copper in the center, down making the liquid touches it

Avoid air bubbles as you can see in this figure:

If it happens, lift the plastic Petre softly and put down again.

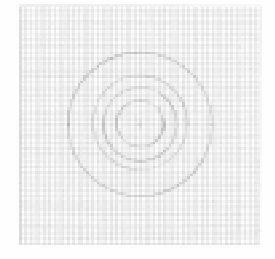
Turn the plastic Petre around in a way that the solution can cover the copper ring and starts to overflow by the “corners”.

Press softly the centre until you can notice that all the plastic Petre is based and the solution excess is out.

Dry the solution that went out with a paper towel.

The end of the copper ring is exposed and it will be the positive electrode (anode), in other words, it will be the direct contact between the solution of $CuSO_4$ and the copper wire in the center of the plastic Petre will be the negative electrode (cathode).

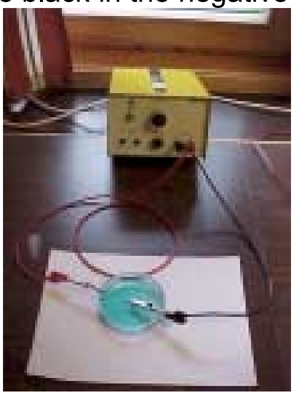
Take the square paper

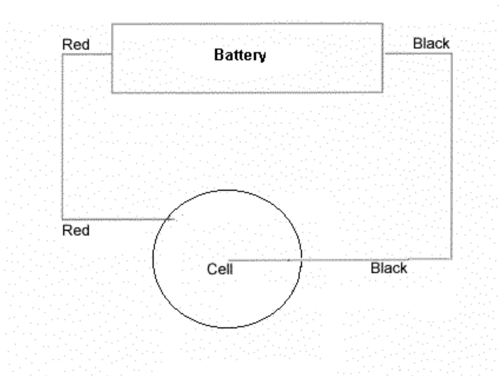


and put under the cell, in a way that the central point of the circles coincides with the central electrode of the cell.

Finally we will connect the cell and we will start the electrolytic deposits.

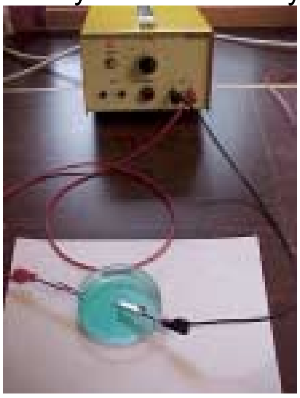
For this you will use a battery and the connections. Remember that the red connection is in the positive pole and the black in the negative pole.



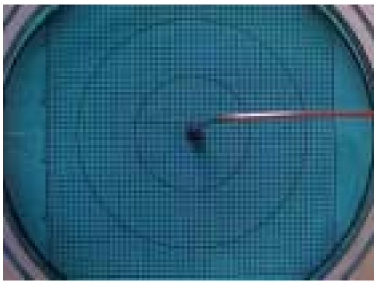


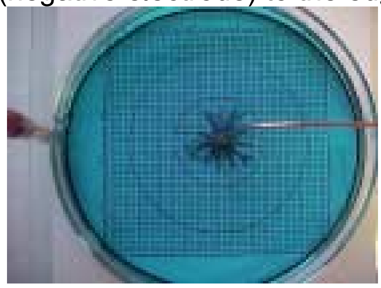
Select 10 volts.

Finally our cell is ready to make the experiment with fractals, so turn on the battery.



From this moment the electrochemical process and the production of electrolytic deposits have started from the centre (negative electrode) to the edges (positive electrode).





You must turn off the battery as soon as the electrolytic deposits are near of the external edge (the best it is to turn off when one particle has crossed the perimeter line of the circle **d**). Don'' allow that to come near of the edge, because in this way there will be a short circuit (without any danger) between the electrodes and the fractal figure that was produced will be damaged.

Take pictures when the process is finished (when you turn off the battery). Use a digital camera and take at least 3 pictures. We will print these images (pictures) and we will use that to count and to classify the squares.

The printed digital photo will look more or less like this:

Accompany patiently the time of producing of the electrolytic deposits that is around 30 minutes.

Results

Take notes and write in the results sheet the times (in minutes) that were used.

	Time (min)
From the starting of the electrolytic deposits until one particle has crossed the perimeter line of the circle **a**	
After crossing **a** and reaches **b**	
After crossing **b** and reaches **c**	
After crossing **c** and reaches **d**	

Attention: The figure that you get from the electrolytic deposits is so fragile. Take a lot of care to move it if you want to keep it.
Who wants to save the results of the experiment must record in a video or take pictures because after some time the figure will be changed due to evaporation of the water, oxidation of the metals etc.

After the electrolytic deposition finishes and you took notes of the results (writing that in the results sheet), we will calculate the fractal dimension of the figure, using the density method.

We advice you make this part in a couple. One notices the figure, counts and classifies the squares and another one takes notes of the values.

For that we will note each 2x2mm square and classify as:

0 empty, without any electrolytic deposition
1 some deposition until 25% of the square's area
2 deposits between 25% and 50% of the square's area
3 deposits between 50% and 75% of the square's area
4 deposits between 75% and 100% of the square's area

The figure is composed of 4 circles:
One circle **a** with radius = 1cm, one **b** with radius = 1,5cm, one **c** with radius = 2 and the another one **d** with radius = 3.

At first count the number of the internal squares to the circle a with its classification from 1 to 4 and take notes in the following table:

		Number of squares in **a**		Number of particles in **a**
1	X		=	
2	X		=	
3	X		=	
4	X		=	
		Total	=	

When the perimeter line of circle **a** is over the inside part of a square, it will be considered inside when 50% or more of the square's area is in the circle **a**.

Multiply, add and get the total of the "number of particles".
Number of particles is between inverted commas because it is a hypothetical particle, a scale of the real particle.

Repeat the same procedure for the circular crown **between the circles b and a**.

		Number of squares in the crown between the circles **b** and **a**		Number of particles in the crown between the circles **b** and **a**
1	X		=	
2	X		=	
3	X		=	
4	X		=	
		Total	=	

As the circle **b** contains **a**, the total of "particles" in **b** will be:

b = a + the crown between the circles b and a = ________

Repeat the same procedure for the **circular crown between the circles c and b**.

		Number of squares in the crown between the circles **c** and **b**		Number of particles in the crown between the circles **c** and **b**
1	X		=	
2	X		=	
3	X		=	
4	X		=	
		Total	=	

As the circle **c** contains **a** and **b**, the total of "particles" in **c** will be:

c = a + b + the crown between the circles c and b = ________

Repeat the same procedure for the **circular crown between the circles d and c**.

		Number of squares in the crown between the circles **d** and **c**		Number of particles in the crown between the circles **d** and **c**
1	X		=	
2	X		=	
3	X		=	
4	X		=	
		Total	=	

As the circle **d** contains **a**, **b** and **c**, the total of "particles" in **d** will be:

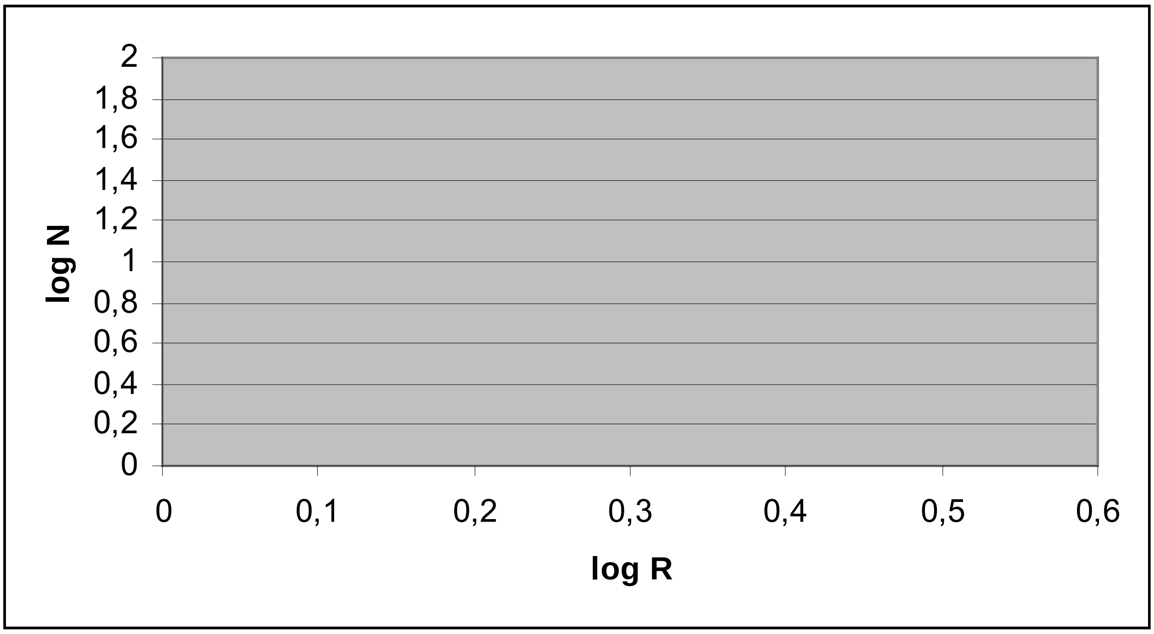
c = a + b + the crown between the circles d and c = ________

Now you will calculate the **Fractal Dimesion**.

We will call total of particles as “N” and the radius as “R”.

Now we can build a graphic log N X log R

Radius (cm)	Total of particles (N)	Log N	Log R
1,0			
1,5			
2,0			
3,0			



As we have seen before, page 14:

$N = (1/r)^D$

Where N is the total of particles and r is the decreasing ration for each of the circles (**a**, **b**, **c** and **d**) and it is 1/R. R is the radius of each circle (**a** = 1cm, **b** = 1,5cm, **c** = 2cm and **d** = 3cm).

Aplying log in both of the sides of the expression $N = (1/r)^D$ we will get:

$\log (N) = \log(1/r)^D$

$\log(N) = D.\log \frac{1}{\frac{1}{R}}$

log N = D. Log R

If we compare with a simple graphic where the expression that represente it is:

(y) = k (x)

So we can have **y** as **logN**, **x** as **logR** and **k** as **D**.

$$D = \frac{\log N}{\log R}$$

So, the fractal dimension will be calculated using the best line between the points in the graphic logN x logR.

The number D can be between 1 and 2, indicating the density of particles from the electrolytic deposition.

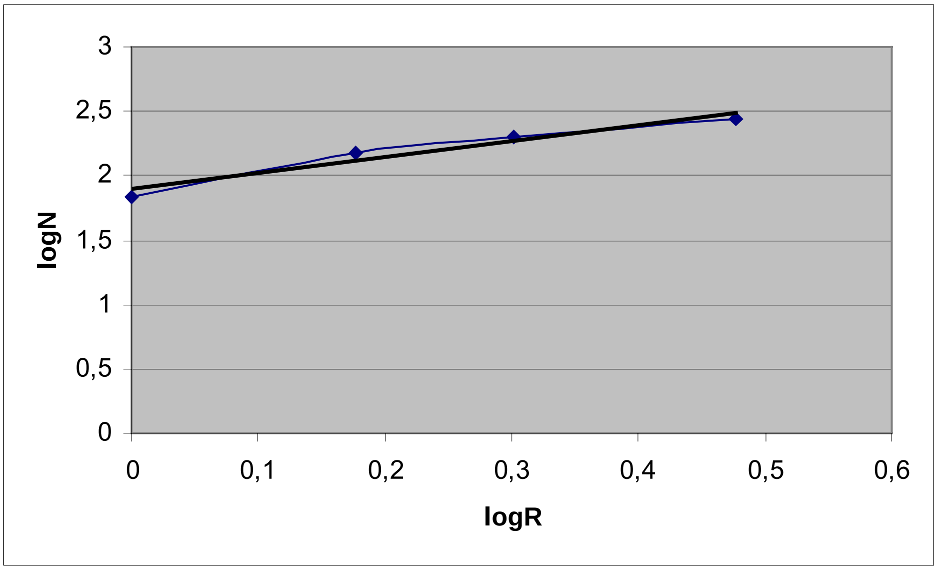
The calculation of the logarithm can be done using a scientific calculator or a computer, in a spreadsheet, for example Excel.

Now we will make an example.

Suppose that the number of particles **a** = 68, **b** = 150, **c** = 199 and **d** = 271, and the radius of the circle **a** = 1cm, **b** = 1,5cm, **c** = 2cm and **d** = 3cm.

Radius (cm)	Total of particles (N)	Log N	Log R
1,0	68	1,8325	0
1,5	150	2,1761	0,1761
2,0	199	2,2988	0,3010
3,0	271	2,4330	0,4771

The graphic with the best line will be:



The best line will have the expression:

And as $D = \frac{\log N}{\log R}$

We will get D = 1,25

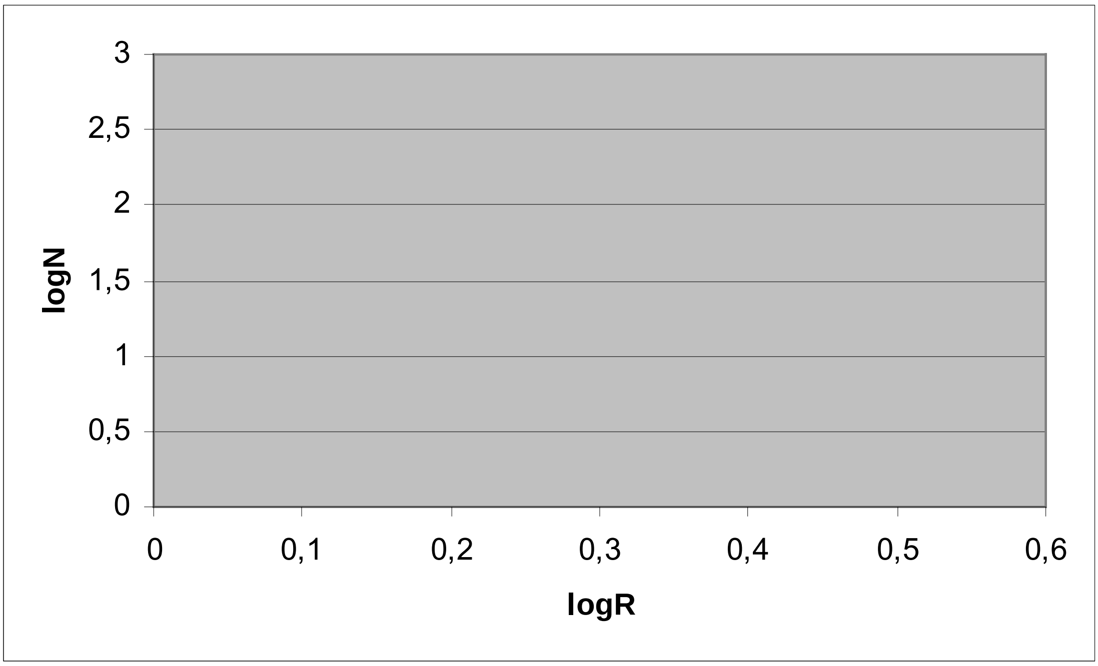
Let's think now about another example that you can make as an exercise. One where the entire circle **d** in the figure of the page 33 is completely filled in, as a plane.

At first you have to count the number of the internal squares to the circle **a** with its classification from 1 to 4 and taking notes in the following table we will get the total of "particles" in **a**.

If you repeat the procedure for the circular crowns between the circles **b** and **a**, **c** and **b** and **d** and **c** we will get the total of "particles" in **b**, **c** and **d**.

Radius (cm)	Total of particles (N)	Log N	Log R
1,0			0
1,5			0,1761
2,0			0,3010
3,0			0,4771

Now you can plot the points, draw the bets line and so calculate the fractal dimension.



$$D = \frac{\log N}{\log R} = \text{------------------} = \text{-----------------}$$

How many dimensions has a plane? ______________________________

References

T. Vicsek, Fractal Growth Phenomena, World Scientific, 1992.

V. Talanquer, G. Irazoque, Fractals: to know, to do, to simulate, Physics Teacher, Vol. 31, 1993.

T. A. Witten and L. M. Sander, Diffusion-Limited Aggregation, a Kinetic Critical Phenomena, Physical Review, Vol. 47, n.19, 1981.

Y. Sawada, A. Dougherty, J. P. Gollub, Dendritic and Fractal Patterns in Electrolytic Metal Deposits, Physical Review Letters, Vol. 56, n. 12, 1986.

S. A. Boggio, R. N. Szente and S. Saadeh, CBCT – Fractais, Lab. de Química, Colégio Bandeirantes, 1998.

B Mandelbrot, Comment j'ai decouvert les fractales, *La Recherche* (1986), 420-424.

W. Fontana, L. W. Buss, The Barrier of Objects: From Dynamical Systems to Bounded Organizations, IIASA, WP-96-27, March 1996.

D. G. Green, Fractals and Scale, Complexity On-line Journal, 1995.

H. F. Jelinek, C. L. Jones. and M.D. Warfel, Is There Meaning in Fractal Analyses?, Complexity International, ISSN 1320-0682, Vol 6, 1998.

M. D. Bayne, The natural geometry of fractals, 11 December 1996.

J. E. Hutchinson, Fractals: A Mathematical Framework, Complexity On-line Journal, Vol2, 1995.

P. Parmananda, Synchronization of chemical systems, Fractal 98, 5th International Multidisciplinary Conference, Kingston University, 1998.

J. Feder, Fractals, Plenum Press, New York and London, November 1989.

Interesting Links

Benoit B. Mandelbrot	http://www.cs.yale.edu/people/faculty/mandelbrot.html
Chaffey High School's FRACTALS on the Web	http://www.chaffey.org/fractals/
Chaos and Fractals: Course Overview	http://millbrook.lib.rmit.edu.au/fractals/subjects.html
Chaos Theory Group	http://www.pd.uwa.edu.au/Physics/Postgrads/Tony_Dixon/chaos_group.html
Complex Systems Virtual Library by themes – Fractals	http://lorenz.mur.csu.edu.au/complex/library/0Fractals.html
Complexity & Artificial Life Research Concept for Self-Organizing Systems	http://members.aol.com/calresco/
Complexity On-line	http://complex.csu.edu.au/complex/index.html
Cynthia Lanius' Lessons A Fractals Lesson – Recognitions	http://math.rice.edu/~lanius/frac/
Dynamical Systems	http://math.bu.edu/DYSYS/dysys.html
Fractal 2000 International Multidisciplinary Conference	http://www.kingston.ac.uk/fractal/
Fractals and Graphics Menu	http://eulero.ing.unibo.it/~strumia/Menu.html
Fractals and scale	http://life.csu.edu.au/complex/tutorials/tutorial3.html
Fractals and their Application to Geometry Models	http://www.fciencias.unam.mx/Graf/fractales/fract_I.html
Fractals as Art	http://www.cs.swarthmore.edu/~binde/fractals/index.html
Fractals on the TI-92	http://members.aol.com/jgrinter/92/fractals.html
Fun with Fractals	http://www.wildfire.com/ag-bin/Fractal
Groupe Fractales	http://www-syntim.inria.fr/fractales/fractales-eng.html
Mandelbrot page of Zs.Zsoldos	http://members.home.net/zzsolt/mandelbrot/mandel.html
The Fractal Microscope	http://www.ncsa.uiuc.edu/Edu/Fractal/Fractal_Home.html
The Fractory An Interactive	http://library.thinkquest.org/3288/

Tool for Creating and Exploring Fractals	
The Natural Geometry of Fractals	http://www.go2net.com/internet/deep/1996/12/11/body.html
The Spanky Fractal Database	http://spanky.triumf.ca/www/spanky.html
What is a fractal?	http://www.dsoe.com/people/hoyle/fractal.html

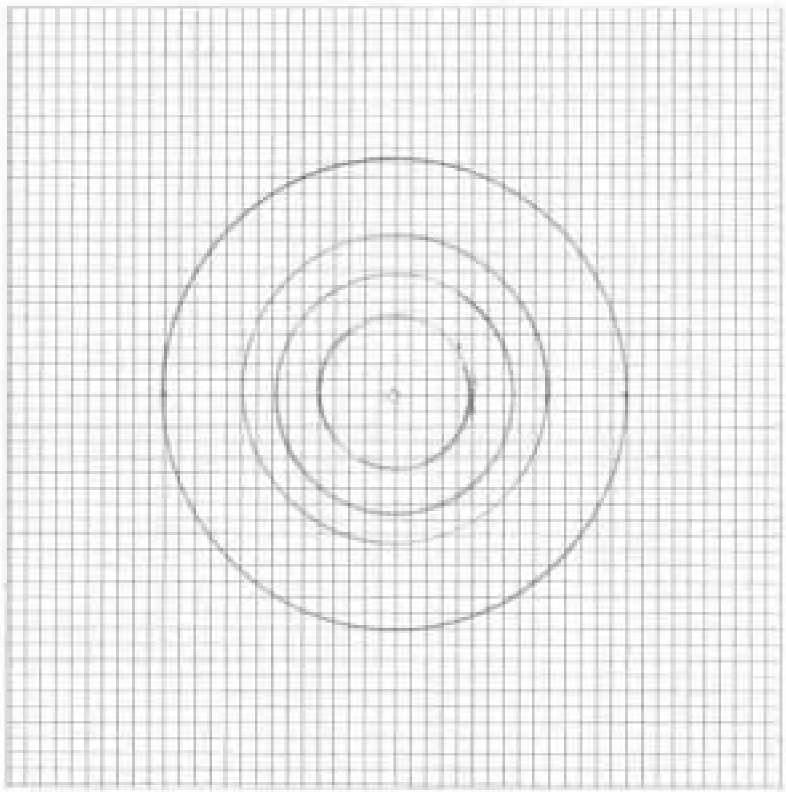
The Fractals

Results Sheet

Date: _______ / _______ / _______

Student's name: ____

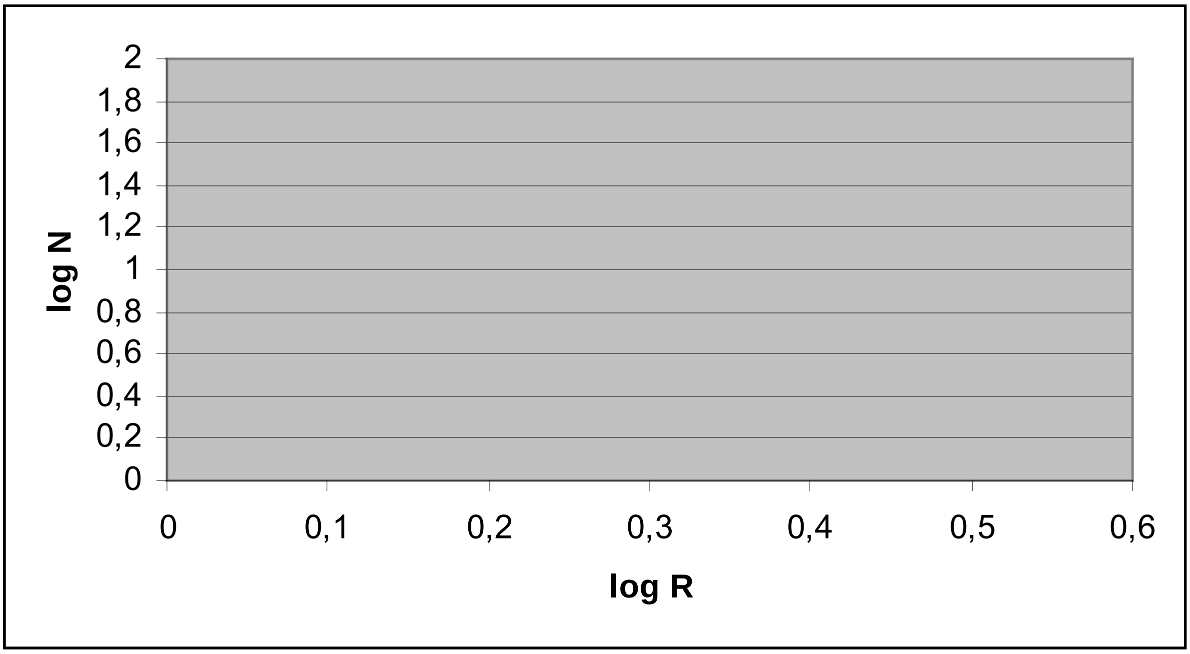
Classroom: ___________________



	Time (min)
From the starting of the electrolytic deposits until one particle has crossed the perimeter line of the circle **a**	
After crossing **a** and reaches **b**	
After crossing **b** and reaches **c**	
After crossing **c** and reaches **d**	

Total of "particles" in **a**	
Total of "particles" in **b**	
Total of "particles" in **c**	
Total of "particles" in **d**	
Fractal dimension	

Graphic:



Observations:

Photo(s)